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Engineering Design Models for Ramjet Efficiency and Lean Blowoff

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Combustion efficiency and lean blowoff semiempirical preliminary design models, validated originally for laboratory axisymmetric flameholders and conventional gas turbine combustors, are extended to dump combustor ramjet geometries. The general efficiency model includes effects of fluid mechanics, spray evaporation, and finite chemical kinetics, and its application to ramjets requires inclusion of choked flow due to the downstream nozzle. For the work reported on here, test data employing liquid fuel injection far upstream of the flameholder are utilized, so that the configuration is modeled as prevaporized and premixed. The model successfully correlates these data, which include variations in combustor geometry, nozzle area ratio, and overall equivalence ratio. The resulting correlation allows the prediction of combustion efficiency for the special case examined to date. Work with the lean blowoff model is limited to comparison of the original model with results presented in the literature.

Nomenclature

A	=	$\pi\rho_2 A' D_3^3/4\dot{m}_a\tau_\eta^*$, with station subscript, cross-sectional area, m^2
A'	=	empirical constant
a	=	coefficient in T_η, usually 0.9
b	=	least-squares-fit y intercept
c	=	empirical constant
D, d	=	diameter, m
E	=	activation energy, cal/gmole
h	=	step height, m
k, k_1	=	empirical constants
L	=	characteristic length, m
L_c	=	combustor (excluding nozzle) length, m
L_n	=	nozzle length, m
L_r	=	reattachment length, m
l_{co}	=	characteristic CO quench length, m
l_{quench}	=	axial CO quench length, m
M	=	Mach number
$\dot{m}_a$	=	airflow rate, kg/s
R	=	gas constant, kcal/mole (or kg) K; least-squares-fit correlation coefficient
$sY\|x$	=	standard error of estimate
T	=	temperature, K
T_η	=	$aT_2 + (1 - a)T_4$
V	=	velocity, m/s
η_c	=	combustion efficiency
ρ	=	density, kg/m^3
τ	=	characteristic time, ms
τ_c	=	combustion time, ms
τ_{co}	=	kinetic time for CO oxidation, ms
τ_{eb}, τ_v	=	droplet evaporation time, ms
τ'_{eb}	=	$(T_{\phi=1}/T_2)\tau_{eb}$
τ_{fi}	=	time associated with fuel injection, ms
τ'_{fi}	=	$(T_{\phi=1}/T_2)\tau_{fi}$
τ_{hc}, τ_{id}	=	ignition delay time, ms
τ'_{hc}	=	$(T_{\phi=1}/T_2)\tau_{hc}$
τ_m	=	mixing time, ms
$\tau_{sl,co}$	=	shear layer quenching time for CO, ms
τ_η, τ_η^*	=	kinetic time for fuel and CO oxidation, ms
ϕ	=	fuel-air equivalence ratio

Subscripts

c, comb	=	combustor
co	=	CO emissions
fi	=	fuel injection
opt	=	optimum
pz	=	primary zone
quench	=	reaction quenching
sl	=	shear layer
η	=	efficiency
2, 3, 4, 5	=	station locations

Introduction

CURRENT designs for the ramjet combustor can suffer from low combustion efficiencies, poor flame stabilization limits, and combustion instability. In liquid-fueled systems, improving these aspects of performance is generally accomplished by varying the number and location of fuel injectors and flame stabilizers, or by adding a swirler in the air inlet. Models of the combustion process, if available and accurate, would reduce the time and cost of the development process. However, finite difference techniques, using state-of-the-art submodels for combustion chemistry, fuel evaporation, and flow turbulence and their interactions, are not presently capable of accurate performance predictions in these complex recirculating flows.

Semiempirical models offer an alternative more useful in the preliminary design and perhaps developmental phases; these focus only on important regions of the combustor flowfield and relate conditions there to inlet conditions and geometry. One such model is the characteristic time model, in use in industry.[1] An alternative but similar approach, developed first with laboratory flameholders and then extended to both gas turbine efficiency and flame stabilization,[2,3] will be developed here for combustion efficiency and lean blowoff limits for dump combustor ramjets.

Proposed Efficiency Model

An efficiency model developed for bluff-body flameholders can be extended to ramjets. Tuttle et al.,[4] using a disc-in-duct

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burner configuration with volatile fuel, treated the shear layer surrounding the bluff-body recirculation zone as the relevant reactor (instead of the entire combustor), and developed an expression for CO emissions as follows:

$$\text{COEI} = \exp(-c\tau_{\text{sl,co}}/\tau_{\text{co}}) \tag{1}$$

where COEI is CO emissions index in grams of CO per kilogram of fuel. The constant c allows Eq. (1) to be written as an equality: Tuttle et al.[4] found $c = 9.45$ based on a least-squares-fit of their data.

For completeness, we modify the model to account for fuel vaporization by adding an evaporative time τ_{eb} as longer fuel evaporation delays observed with jet fuel increased CO emissions,[4] so that

$$\text{COEI} = \exp[-c\tau_{\text{sl,co}}/(\tau_{\text{co}} + k\tau_{\text{eb}})] \tag{2}$$

where k is determined by fitting experimental data to Eq. (2) and is necessary as characteristic times are order of magnitude estimates of the actual times.[4]

Following Blazowski and Henderson,[5] combustion inefficiency is taken as proportional to a weighted sum of CO and HC emissions. Schmidt and Mellor[6] found that CO and HC quenched in essentially the same region of the disc-and-duct combustor flowfield and were proportional to one another. Thus, the following equation for efficiency is proposed:

$$\eta_c = 1 - \exp[-A'\tau_{\text{sl,co}}/(\tau_\eta^* + k\tau_{\text{eb}})] \tag{3}$$

Here, τ_η^* is the characteristic time for the combination of CO and HC oxidation in lean mixtures,[2] and the value of A' is also empirically derived. For ramjets, which generally exhibit lower efficiencies than turbojets, we retain the exponential Eq. (3). Tuttle et al.,[4] Schmidt and Mellor,[6] and Leonard and Mellor[2] expanded the exponential in a power series in characteristic time ratio, and showed that retaining only the linear term was adequate for the disc-in-duct and for turbine engines.

The postulated efficiency model will now be examined in several stages, leading to an equation directly comparable to that of Gallagher et al.,[1] who also use characteristic times to describe ramjet performance. First, step height or flameholder geometry effects will be considered. Then, the scale in $\tau_{\text{sl,co}}$ will be modified to account for finite combustor length. Next, τ_η^* will be defined. Finally, the appropriate gas dynamics corresponding to the presence of a choked nozzle at the exit of the combustor will be modeled, thus coupling both $\tau_{\text{sl,co}}$ and τ_η^*.

Flameholder Geometry Effects

Let us examine how Eq. (3) includes combustor and geometry effects, first ignoring combustor length and exhaust nozzle effects. The shear layer residence time, as defined by Plee and Mellor's[7] work with a tube-and-disc combustor, which involved a sudden expansion at the end of a fuel preparation tube, is

$$\tau_{\text{sl,co}} = (D_3 - D_2)/V_2 \propto (D_3 - D_2)D_2^2 \tag{4}$$

for an axisymmetric tube-and-disc burner specialized here to the dump burner geometry shown in Fig. 1. Further algebra results in the following relationship:

$$\tau_{\text{sl,co}} \propto D_3^3(2h/D_3)(1 - 2h/D_3)^2 \tag{5}$$

where h is defined as $(D_3 - D_2)/2$. In this expression, as h approaches the two limits of 0 and $D_3/2$, $\tau_{\text{sl,co}}$ and efficiency approach the correct limit of zero. For the following analysis fuel injection far upstream of the step is assumed, making heterogeneous effects associated with fuel vaporization negligible. This restricts the results to a prevaporized premixed ramjet (PPRJ).

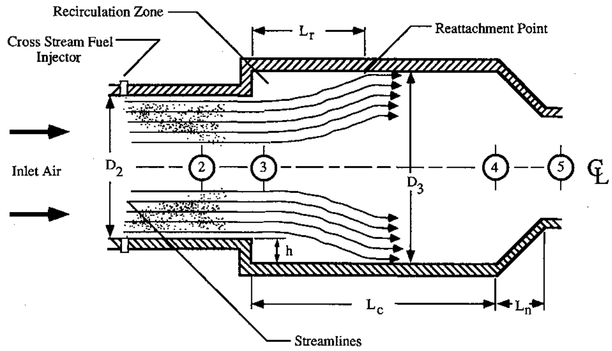

Fig. 1 Liquid fuel ramjet schematic.

Assuming negligible vaporization effects, the equation for efficiency becomes

$$\eta_c = 1 - \exp[-A(2h/D_3)(1 - 2h/D_3)^2] \tag{6}$$

where A is proportional to D_3^3 and inlet air density, and inversely proportional to mass flow rate and τ_η^*. Note that taking the derivative of $\tau_{\text{sl,co}}$ with respect to h in Eq. (5) and solving for maximum efficiency results in $h_{\text{opt}} = D_3/6$. The existence of an optimum step height has been observed, at least for liquid-fueled systems.[8]

Combustor Length Effects

The previous model, developed for the tube-in-duct combustor, does not explicitly account for combustor length effects. However, as Stull et al.[8] experimentally determined and as Gallagher et al.[1] predict, for ramjets efficiency increases with L_c. Mellor[9] proposed the following definition of $\tau_{\text{sl,co}}$ for a gas turbine combustor:

$$\tau_{\text{sl,co}} = l_{\text{co}}/V_2 = (1/d_{\text{comb}} + 1/l_{\text{quench}})^{-1}/V_2 \tag{7}$$

In gas turbine engines, l_{quench} is defined by Mellor and Washam[10] to be the downstream axial distance from the fuel injector tip to where the station-average equivalence ratio falls below 0.2 due to air penetration jets. Thus, Eq. (3) can be written in terms of this definition of $\tau_{\text{sl,co}}$ as

$$\eta_c = 1 - \exp[-A'l_{\text{co}}/V_2(\tau_\eta^* + k\tau_{\text{eb}})] \tag{8}$$

For ramjet combustors, with higher overall equivalence ratios (and lower efficiencies), CO oxidation continues until the products reach the nozzle, which may define quench length equal to combustor length. Then l_{co} is defined as

$$l_{\text{co}} = (1/D_3 + 1/L_c)^{-1} \tag{9}$$

or considering the sudden expansion (dump combustor) configuration, where $D_3 - D_2$ is the radial characteristic length

$$l_{\text{co}} = [1/(D_3 - D_2) + 1/L_c]^{-1} = (1/2h + 1/L_c)^{-1} \tag{10}$$

Further algebra results in

$$\tau_{\text{sl,co}} \propto D_3^3(2h/D_3)(1 - 2h/D_3)^2/[1 + (2h/D_3)(D_3/L_c)] \tag{11}$$

Note that the new term appearing in the denominator of Eq. (11) suggests the optimal step height becomes a function of L_c/D_3. Assuming as before the prevaporized, premixed ramjet (PPRJ), efficiency is a function of combustor geometry as follows:

$$\eta_c = 1 - \exp\{-A(2h/D_3)(1 - 2h/D_3)^2 \times [(L_c/D_3)/(L_c/D_3 + 2h/D_3)]\} \tag{12}$$

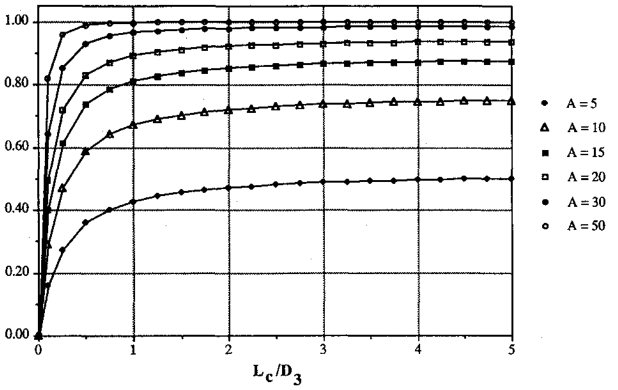

Fig. 2 Calculated combustor efficiency vs PPRJ combustor length to diameter ratio ($h = D_3/6$) using the gas turbine combustor definition for shear layer residence time [Eq. (12)].

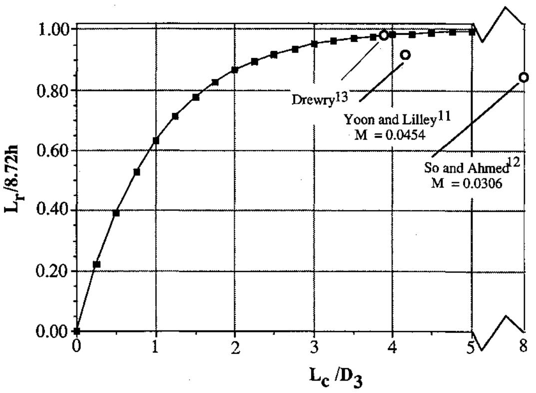

Fig. 3 Proposed relation between reattachment length and combustor length [Eq. (13)], and average of Drewry's measurements.[13]

Figure 2 shows a graph of this expression for various values of A and step height equal to $D_3/6$.

An alternative approach to include combustor length effects can be formulated as follows. In both the tube-in-disc and disc-and-duct configurations, the flameholder dimension is used as proportional to recirculation zone length.[4,7] In the sudden expansion flow of Fig. 1, L_r replaces recirculation zone length.[7] Thus, $D_3 - D_2$ (or $2h$) is written in Eq. (4) as a replacement for the (unknown) L_r. Yoon and Lilley[11] found that for nonswirling cold flow over a sudden expansion where $D_3/D_2 = 1.5$ or 2, with no downstream blockage, L_r is equal to about 8 step heights, with a Mach number of 0.045. Similarly, So and Ahmed[12] found $L_r/h = 7.4$ with a Mach number of 0.03. Drewry,[13] using his cold flow results for an axisymmetric configuration ($L_c/D_3 = 3.93$, $D_2/D_3 = 0.65$, $L_n/D_3 = 0.39$, where L_n is axial nozzle length, and $D_3 = 9.75$ cm) with a choked nozzle, and varying D_5^* from 5.08 to 7.62 cm, found values of L_r ranging from $7.5h$ at an inlet Mach number M_2 of 0.42 to $9.0h$ at $M_2 = 1.0$. Combined with other results found in the literature, he brackets the relationship between reattachment length and step height by two linear equations, $L_r = 7.9h$ and $L_r = 9.2h$, where the first equation is applicable to lower inlet flow Mach numbers $M_2 \approx 0.5$, and the second to higher values $M_2 \approx 0.9$.

With a downstream nozzle as in a short, compact ramjet, L_r is expected to become a function of L_c. Because no functional relationship between L_r and L_c is available, the following in the absence of a swirler is proposed, using the average of Drewry's[13] two equations ($L_r = 8.55h$) at his $L_c/D_3 = 3.93$ as a defining point (see Fig. 3):

$$L_r = 8.72h[1 - \exp(-L_c/D_3)] \tag{13}$$

Equation (13), i.e., the specific value of 8.72, could be adjusted accordingly to reflect changes in L_r with reacting flow.

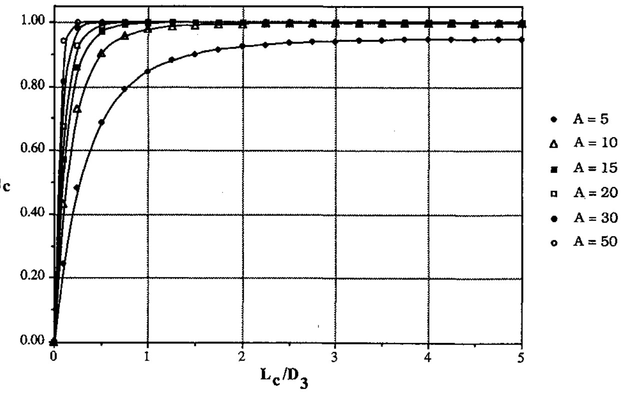

Fig. 4 Calculated combustor efficiency vs PPRJ combustor length to diameter ratio ($h = D_3/6$) using the reattachment length function [Eq. (13)].

In either case, the exponential gives the correct qualitative function for L_r as a function of combustor length, and the definition for $\tau_{sl,co}$ becomes

$$\tau_{sl,co} = L_r/V_2 = 8.72h[1 - \exp(-L_c/D_3)]/V_2$$

$$\tau_{sl,co} \propto 4.36D_3^3(2h/D_3)(1 - 2h/D_3)^2 \times [1 - \exp(-L_c/D_3)] \tag{14}$$

Again assuming negligible evaporation effects, Eq. (3) becomes

$$\eta_c = 1 - \exp\{-4.36A(2h/D_3)(1 - 2h/D_3)^2 \times [1 - \exp(-L_c/D_3)]\} \tag{15}$$

Figure 4 shows this efficiency equation for $h = D_3/6$.

Although Eqs. (12) and (15) predict the same trends for efficiency, a comparison of Figs. 2 and 4 for any value of A at $L_c/D_3 = 5$ reveals a significant difference in predicted efficiency. Following Tuttle et al.,[4] who also studied toothed-disc flameholders, and in these cases defined l_{co} as the inverse of the empirically weighted average of the reciprocals of the two dissimilar characteristic lengths (tooth width and disc diameter), we can use an empirical parameter k_1 in Eq. (10) to weight one of the also dissimilar PPRJ characteristic dimensions (step height or combustor length) in l_{co}:

$$l_{co} = 1/(k_1/2h + 1/L_c) \tag{16}$$

Given experimental data, k_1 can be determined such that Eqs. (12) and (15) yield comparable results for specific values of A and L_c/D_3.

The present efficiency model for ramjet combustors is given by Eq. (8). For a PPRJ with $\tau_{eb} = 0$ by definition

$$\eta_c = 1 - \exp(-A'l_{co}/V_2\tau_\eta^*) \tag{17}$$

Possible definitions of l_{co} have been discussed above, leaving the definition of τ_η^* to be addressed.

Effects of Operating Conditions

Schmidt and Mellor[6] correlated kinetic effects on efficiency for various fuels in a disc-stabilized combustor as follows:

$$\tau_\eta = 0.01 \exp(E/RT_\eta) \tag{18}$$

where E is the activation energy for lumped HC and CO oxidation (chosen empirically as 4500 cal/g mole). Schmidt and Mellor[6] found that for a disc-in-duct burner and for gas turbine combustors, as overall equivalence ratio increases, exhaust emissions of CO and HC decrease. They then include equivalence ratio effects on efficiency by incorporating the burned gas temperature, so that kinetic temperature dependence is a function of the inlet and exit gas temperatures. Schmidt and Mellor[6] found best correlation for their disc-stabilized flames with $T_\eta = 0.9T_2 + 0.1T_4$. The temperature coefficients for a ramjet may not be weighted the same, as the higher overall equivalence ratios result in higher downstream temperatures, allowing CO oxidation to continue.

Leonard and Mellor[2] correlated turbine combustor efficiency data, and redefined the HC and CO oxidation time for lean primary zones as τ_η^*, where

$$\tau_\eta^* = \tau_\eta = 0.01 \exp(E/RT_\eta) \quad \text{if} \quad \phi_{pz} > 1.0 \tag{19}$$

$$\tau_\eta^* = \tau_\eta/\phi_{pz} \quad \text{if} \quad \phi_{pz} < 1.0 \tag{20}$$

Here, ϕ_{pz} is (average) primary zone equivalence ratio.

Nozzle Effects

Ramjet combustor models must account for a choked nozzle downstream of a duct with heat addition. For a given nozzle area ratio, the nozzle entrance Mach number is essentially constant. Any increase in heat addition (through changes in either ϕ for a given geometry or L_c/D_3 for a given ϕ) will result in decreased combustor inlet Mach number and velocity. Thus in actuality τ_η^* and $\tau_{sl,co}$ are coupled in a ramjet, unlike a turbojet. This coupling can be accounted for using one-dimensional gas dynamics analysis.

Experimental data from Craig et al.[14,15] are available for comparison with the PPRJ model. These workers, in studying axial dump combustor efficiencies, injected JP-4 far upstream (1.35 m) of the step to obtain a simulated PPRJ, allowing the fuel to vaporize and mix with the inlet air before reaching the step. Craig et al.,[15] who investigated the effect of percent flameholder blockage on efficiency and pressure losses, studied PPRJ efficiency with a dump combustor geometry similar to Fig. 1, with D_2 = 15.24 cm, D_3 = 30.48 cm, L_c/D_3 = 3 and 4.5, and a nozzle length of 30.48 cm. Craig et al.,[14] who were attempting to correlate combustor performance with nonreacting flowfield results, also studied PPRJ performance, but with D_2 = 10.16 cm, D_3 = 15.24 cm, L_c/D_3 = 1, 2, and 3, and a 7.62-cm-long nozzle.

The calculation of characteristic times is a straightforward procedure, provided that combustor geometry and inlet conditions are known. Unfortunately, all of the necessary information to make these calculations is not always measured in ramjet tests. To use these experimental PPRJ data for model correlation, it is necessary to calculate the combustor inlet velocity V_2, and the combustor exit temperature T_4 from available information. A one-dimensional compressible flow approach is used to obtain these values. Figure 5 shows known parameters at each station of the flow and assumptions made to calculate the unknowns required for the analysis. A detailed description of calculation steps is found in Prior.[17]

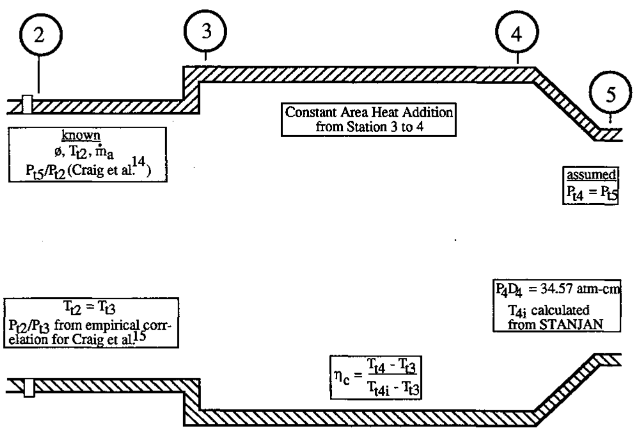

Fig. 5 Ramjet combustor schematic showing available information and assumptions made to calculate V_2 and T_4. STANJAN is the thermochemical equilibrium code (Reynolds[16]) that was used in the analysis.

Lean Blowoff Model

A lean blowoff model developed with characteristic times is also applicable to ramjets. Plee and Mellor[7] correlate nearly 400 lean blowoff datum points using characteristic times generally equivalent to those of Gallagher et al.[1] with slightly different definitions and modifications, but focus on the shear layer originating at the step lip, as in the combustion efficiency model just discussed. These results are shown in Fig. 6. The relations between the characteristic times from Plee and Mellor[7] on the axes in Fig. 6 and those of Gallagher et al.[1] are as follows:

$$\tau_{sl,co} + 0.12\tau'_{fi} \rightarrow \tau_m \tag{21}$$

$$\tau'_{hc} + 0.011\tau'_{eb} \rightarrow \tau_{id} + \tau_v \tag{22}$$

where τ_c is

$$\tau_c = \tau_v + \tau_{id} + \tau_m \tag{23}$$

and τ_v is the vaporization time and τ_{id} is the ignition time. Plee and Mellor[7] found it necessary to modify their shear layer time to include fuel penetration through the shear layer τ_{fi} for the disc-in-duct combustor burning heavy fuels. τ_{fi} may also represent cross stream liquid fuel injection effects when data from liquid fuel ramjets are examined in the future. τ'_{hc} is the hydrocarbon ignition delay time, τ'_{eb} is related to the droplet lifetime previously described (see Nomenclature), and the coefficients are empirically obtained.

The correlation in Fig. 6 yields the following lean blowoff limit, assuming a negligible y intercept:

$$\tau_{sl,co} + 0.12\tau'_{fi} = 2.12(\tau'_{hc} + 0.011\tau'_{eb}) \tag{24}$$

Substituting Eqs. (21) and (22) into this limit allows rewriting in terms of the characteristic times used by Gallagher et al.[1]:

$$\tau_m = 2.12(\tau_{id} + \tau_v) \tag{25}$$

Thus, Plee and Mellor's[7] lean blowoff limit can be superimposed on the results of Gallagher et al., as shown in Fig. 7, by dividing Eq. (23) by τ_m

$$\tau_c/\tau_m = 1 + (\tau_{id} + \tau_v)/\tau_m \tag{26}$$

and substituting the empirical relation Eq. (25):

$$\tau_c/\tau_m = 1 + 1/2.12 = 1.47 \tag{27}$$

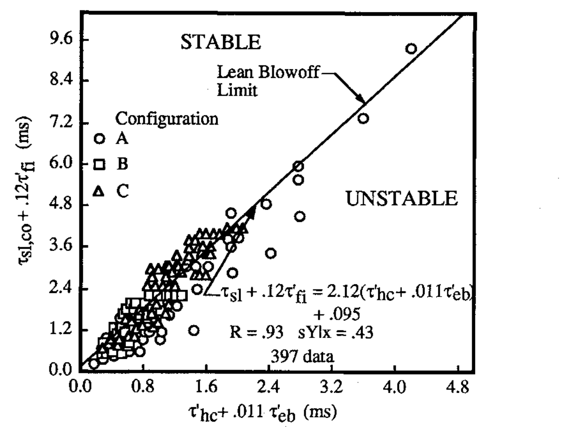

Fig. 6 Lean blowoff model (after Plee and Mellor[7]). Configurations A, B, and C are different flameholder geometries and fuel insertion methods.[7]

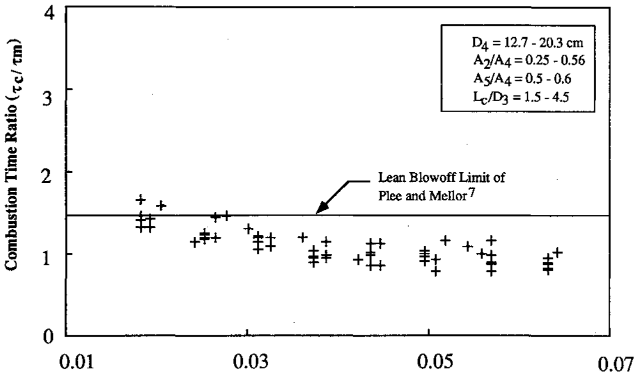

Fig. 7 Combustion time ratio vs fuel/air ratio (after Gallagher et al.[1]).

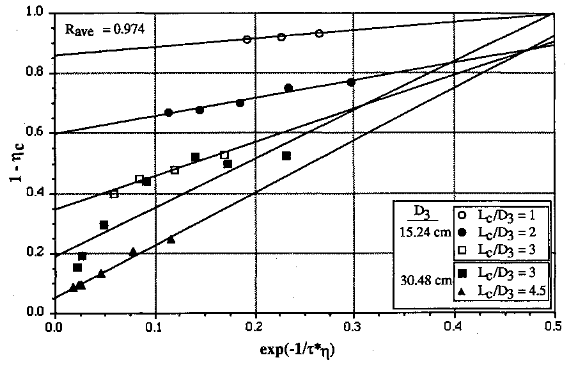

Fig. 8 Inefficiency vs $\exp(-1/\tau_\eta^*)$ for five combustor geometries. Here $T_\eta = 0.9T_2 + 0.1T_4$, and R_{ave} is the average of the correlation coefficients for the five least-square-fits shown as solid lines.

Represented as a horizontal line in Fig. 7, this limit corresponds to the lack of ramjet operating points, which indicates lean blowoff. Thus, in general terms the results of Gallagher and coworkers are consistent with the analysis of Plee and Mellor. Given appropriate values of inlet velocity, combustor geometry, inlet temperature, fuel type, etc., lean blowoff limits using Eq. (24) can be calculated in terms of explicit fuel/air ratios.

Prevaporized, Premixed Ramjet Efficiency Correlations

Validation of the efficiency model [Eq. (17)] is best accomplished if the equation is rewritten as a linear expression:

$$\ln(1 - \eta_c) = -A'\tau_{\text{sl,co}}/\tau_\eta^* = -A'l_{\text{co}}/V_2\tau_\eta^* \qquad (28)$$

A' is then the slope of the least-squares-linear fit of the data. For a given geometry, kinetic effects through τ_η^* will be examined first.

Inefficiency vs $1/\tau_\eta^*$ for each of the five PPRJ configurations is shown in Fig. 8. Recall from Eqs. (19) and (20) the definition of τ_η^* and its dependence on ϕ_{pz}. For ramjets, without secondary air injection as in turbojets, ϕ_{pz} is taken as ϕ. For the data shown in Fig. 8, $T_2 = 550$ K, and as ϕ is increased for a given combustor geometry, both η_c and T_4 increase.

Figure 8 shows that the data do follow the expected trends of increasing efficiency with equivalence ratio, and least-squares-linear fits of the data for a given geometry are represented by straight curves through the data. The data for the fourth combustor ($L_c/D_3 = 3$, $D_3 = 30.48$ cm) deviate from the relationship demonstrated by the other four geometries, and are not monotonic with $\exp(-1/\tau_\eta^*)$.

Other dependencies of T_η on T_2 and T_4 were examined via the coefficient a in $T_\eta = aT_2 + (1 - a)T_4$ to see if correlations superior to those in Fig. 8 (with $a = 0.9$) resulted, particularly for combustor number four. The optimal value of a was 0.95, but the improvement in average correlation coefficient was so marginal (0.975 rather than 0.974 for those fits shown in Fig. 8), that the $a = 0.9$ was retained.

Equation (28) is evaluated twice to account for both definitions of l_{co}. Rows 1 and 2 of Table 1 present the value of A', the y-intercept b, the correlation coefficient R, and the corresponding standard deviation of the correlation ($sY|x$) for each case. Table 1 shows that the correlation qualities are independent of the characteristic CO quench length definition. Although the two values of A' are different, the correlation coefficients are the same. An explanation of this is found in Figs. 2 and 4, which show that different numerical values of the constant A (or A') are required to produce identical correlations for the two definitions of l_{co} investigated. A representative efficiency correlation corresponding to Eq. (10) for l_{co} is shown in Fig. 9 in terms of Eq. (17) rather than Eq. (28). If combustor four exhibiting scatter in Fig. 8 is excluded from the correlation, R improves to 0.991 with $sY|x = 0.06$ for the Eq. (10) definition of l_{co}.

In an effort to improve the correlations, empirical constants can be used in the two definitions of l_{co}. As discussed previously, Tuttle et al.[4] defined l_{co} as the inverse of the weighted average of the reciprocals of two dissimilar characteristic lengths [Eq. (16)]. For the reattachment length definition of l_{co} [Eq. (13)], it was assumed previously that reattachment length continued to increase with combustor length until a maximum L_r was reached at $L_c/D_3 = 4$ or 5 (Fig. 3). However, L_r may reach a maximum at smaller L_c/D_3, in which case an empirical factor is used in Eq. (13)

$$l_{\text{co}} = L_r = 8.72h[1 - \exp(-gL_c/D_3)] \qquad (29)$$

where g is empirically determined from the best fit of the experimental data.

With these two definitions of l_{co}, Eq. (28) is again utilized to determine new values of A' and R for the correlations. The statistics for the optimum correlations are presented in Table 1. For $l_{\text{co}} = 1/(k_1/2h + 1/L_c)$, the best occurs with $k_1 = 0$, but is misleading as l_{co} reduces to L_c in this case. This definition of l_{co} does not account for step height, which is expected to affect efficiency to a greater extent at values of h/D_3 closer to 0 and 0.5, as discussed previously. Equation (29) reduces to $8.72g(h/D_3)L_c$ for $g = 0.001$, implying that $\tau_{\text{sl,co}}$ is a function of step height-to-diameter ratio and L_c. Both of these results verify that the experimental range of h/D_3 variation (0.167–0.25) is too limited with the present data to verify the postulated dependencies of both definitions of l_{co} on step height. However, testing with values of h/D_3 outside this range may not be possible due to flame extinction: a curve of blowoff equivalence ratio vs h/D_3, derived from the model of Plee and Mellor,[7] shows the leanest limit occurs at the same optimum range of h/D_3, and blowoff equivalence ratio approaches infinity as h/D_3 approaches either zero or one-half, the inverse dependence of combustion efficiency.

To this point it has been assumed that CO quenching occurs at the combustor exit plane. However, for shorter ramjet combustors, CO oxidation may continue into the nozzle if the combustion gas temperature is sufficiently high. Gallagher et al.,[1] in developing their combustor efficiency time model, chose to use the combustor length plus the nozzle length $L_c + L_n$ in their definition of characteristic residence time.[18] For the efficiency model studied here, nozzle length can be incorporated into the definition of l_{quench}, because ϕ remains in excess of 0.2[10]:

$$l_{\text{co}} = [k_1/2h + 1/(L_c + L_n)]^{-1} \qquad (30)$$

Nozzle length is not included in the reattachment length definition of l_{co}, because L_r is not expected to be a function of nozzle length, but of downstream nozzle position. Table 1

Table 1 Parameters examined to optimize efficiency correlations for data of Craig et al.[11,12]

Variation	A'	b	R	$sY\|x$	Optimum	Figure
	$\ln(1-\eta_c) = -A'l_{co}/V_2\tau_\eta^* + b$					
$l_{co} = [1/2h + 1/L_c]^{-1}$	1.1355	0.4015	0.940	0.0866	——	9
$l_{co} = 8.72h[1 - \exp(-L_c/D_3)]$	0.2249	0.3016	0.937	0.0803	——	——
$l_{co} = [k_1/2h + 1/L_c]^{-1}$	0.1188	0.2186	0.991	0.041	$k_1 = 0$	——
$l_{co} = 8.72h[1 - \exp(-gL_c/D_3)]$	50.341	0.0292	0.988	0.052	$g = 0.001$	——
$l_{co} = [k_1/2h + 1/(L_c + L_n)]^{-1}$	1.0099	0.2149	0.926	0.100	$k_1 = 1$	10
$l_{co} = [k_1/2h + 1/(L_c + L_n)]^{-1}$	0.0906	0.1199	0.986	0.0519	$k_1 = 0$	——
$l_{co} = (1/2h + 1/L_c)^{-1}$	0.3878	0.0291	0.951	0.074	$a = 0.74$	9
$l_{co} = 8.72h[1 - \exp(-L_c/D_3)]$	0.0789	−0.0095	0.949	0.0717	$a = 0.74$	——

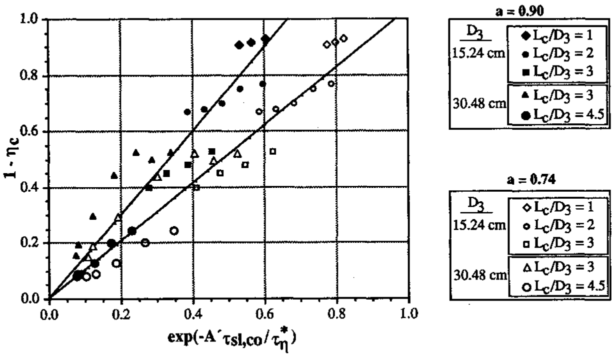

Fig. 9 PPRJ inefficiency model using $l_{co} = 1/[1/2h + 1/L_c]$ for two values of a in T_η.

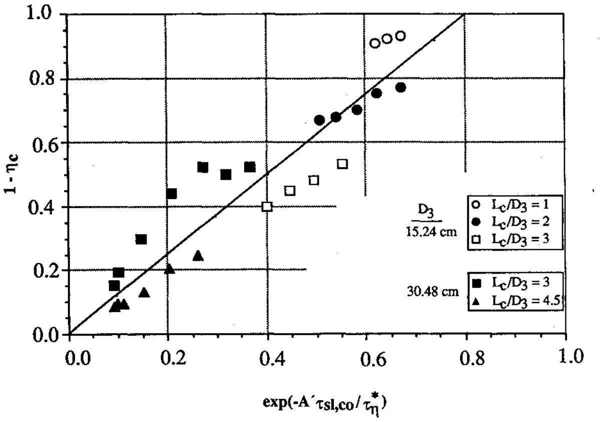

Fig. 10 PPRJ inefficiency model using $l_{co} = 1/[1/2h + 1/(L_c + L_n)]$.

shows that the correlation coefficient using Eq. (30) is essentially the same as the value of R determined using only L_c in the definition of l_{co}, but the standard deviation has increased 15%, implying that the inclusion of nozzle length in the model will not improve the correlation, but will in fact be a detriment (see Fig. 10). An attempt to optimize the correlation using Eq. (30) results in $k_1 = 0$ as before, with the corresponding information provided in Table 1.

A value of $a = 0.9$ in T_η has been used for all of the correlations discussed to this point in Table 1, based on the results shown in Fig. 8, where V_2 was assumed constant for various values of ϕ. However, for ramjets geometry ($\tau_{sl,co}$) and operating conditions (τ_η^*) are as noted coupled due to the presence of the choked nozzle (V_2 and T_4 both depend on ϕ). Therefore, Eq. (28) was reoptimized with respect to a in T_η for both definitions of l_{co}, and the results are found in Table 1. With optimal $a = 0.74$, the correlation quality increases and the standard deviation decreases compared to values using $a = 0.9$ for identical l_{co} definitions. Of more significance, the y intercept b for each correlation becomes less than one standard deviation, in agreement with the original model Eq. (17) for which $b = 0$ (so that $e^b = 1$). Figure 9, which also shows the efficiency correlation corresponding to Eq. (10) for l_{co} with $a = 0.74$, shows that only the slopes in Eq. (17) have changed, without significant decrease in scatter.

Conclusions

Previous characteristic time efficiency models developed for laboratory flameholders and gas turbine combustors include expected trends for a PPRJ geometry (premixed gaseous fuel and air, but finite ignition delay). A corresponding lean blowout model is generally consistent with data reported by Gallagher et al.[1] Applying the efficiency model to PPRJ data from Craig et al.[14,15] has been successful, once the inlet velocity has been determined from the available experimental data. In $\tau_{sl,co}$, two definitions of characteristic length, one a function of two dissimilar lengths (step height and combustor length), and the other based on the reattachment point of the recirculation zone, have been used to model the data with equal success. Weighting of the two dissimilar lengths in the definition of l_{co} improves the model, as does varying the shape of the L_r vs L_c/D_3 relationship in Fig. 3.

Based on the limited datum set for PPRJs available in Craig et al., two preliminary design equations for combustion efficiency are recommended: either uses Eq. (18) and (20) with $T_\eta = 0.9T_2 + 0.1T_4$ to define kinetic effects, but two choices for characteristic length l_{co} remain. For that based on gas turbine correlations, Eq. (10), the efficiency equation is

$$(1 - \eta_c) = 1.49 \exp(-1.136\tau_{sl,co}/\tau_\eta^*) \qquad (31)$$

and for l_{co} defined in terms of reattachment length, Eq. (13)

$$(1 - \eta_c) = 1.35 \exp(-0.225\tau_{sl,co}/\tau_\eta^*) \qquad (32)$$

Alternative and equally good correlations with near-unity coefficients on the exponential term can be obtained from the information presented in Table 1.

Additional PPRJ data are required with more substantial variations in step height to combustor diameter ratio to ascertain its proper inclusion in the model proposed here. Further data with T_2 variations would better test the current definition of τ_η^*, and effects of inlet swirlers and flameholders in addition to the sudden expansion should be added to the model as well.

The eventual goal of the modeling is the liquid-fuel ramjet (LFRJ), for which τ_{eb} in Eq. (3) is nonzero. However, most of the previous characteristic time modeling has been performed for the case of sufficient liquid fuel injection into the shear layer so that it will be stoichiometric, as is design practice for conventional gas turbine combustors, and as was found with the laboratory flameholders burning volatile fuels. In such cases model equations presented above apply, with drop evaporation time nonnegligible. Thus, combustion efficiency will decrease and lean blowoff equivalence ratio will increase as drop lifetimes increase, and PPRJ performance, evaluated at an overall equivalence ratio of unity, represents the standard of comparison for this type of LFRJ. Prior to accounting for nonstoichiometric shear layers, as discussed by Gallagher et al.,[1] due to cross-stream liquid fuel injection, the simplest LFRJ configuration to model next is that with stoichiometric shear layers.

As noted previously, Plee and Mellor[7] and Leonard and Mellor[2] introduced a fuel injection characteristic time τ_{fi} to model fuel penetration effects with heavy fuels in lean blowoff. As seen from Eq. (24), this characteristic time enhances stabilization, as it adds to the shear layer mixing time. Choudhury[19] observed similar effects with transverse gas injection in dump burner configurations. Cross-stream injected LFRJs represent the logical consequent sophistication in the modeling, to be followed with model development for a LFRJ with staged (both cross stream and shear layer injection) combustion zones. In this last case fuel flow split to the two injection locations becomes a new design variable for optimization (and perhaps active control) of performance.

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